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MECHANICS.

A Treatise on Analytical Statics. With numerous Examples. Vol. I. By Edward John Routh, Sc.D., LL.D., F.R.S., Hon. Fellow of Peterhouse, Cambridge; Fellow of the Senate of the University of London. (London: Macmillan and Co., 1891.)

The Elementary Part of a Treatise on the Dynamics of a System of Rigid Bodies. Being Part I. of a Treatise on the Whole Subject. With numerous Examples. By the Same. (London: Macmillan and Co., 1891.)

WITH these two volumes the mathematical student is completely equipped for the course of Analytical Mechanics, as required for Part I. of the Cambridge Mathematical Tripos.

A second volume is promised of the "Analytical Statics," to cover the parts in Attraction, Astatics, and the Bending of Beams; and this, in conjunction with Part II. of the "Dynamics," will complete his library for the second part of the Mathematical Tripos, according to present regulations.

The great feature of these works is the very complete collections of examples which the author has brought together with great labour, and enriched with many of his own invention, fit to rank among the theorems of the science, rather than as mere problems.

The author is of the opinion that in order to learn Mechanics it is essential to the student to work many examples, taken as far as possible from questions that have actually arisen.

In this opinion he agrees with Fourier, who says:—

> "L'étude approfondie de la nature est la source la plus féconde des découvertes mathématiques. Non seulement cette étude, en offrant aux recherches un but déterminé, a l'avantage d'exclure les questions vagues et les calculs sans issue; elle est encore un moyen assuré de former l'Analyse elle-même," &c.

This is an opinion, however, that has always divided mathematicians into rival camps, and we find Jacobi remonstrating with these words of Fourier by retaliating:—

> "Il est vrai que M. Fourier avait l'opinion que le but principal des mathématiques était l'utilité publique et l'explication des phénomènes naturels; mais un philosophe comme lui aurait dû savoir que le but unique de la science c'est l'honneur de l'esprit humain; et que sous ce titre, une question de nombres vaut autant qu'une question du système du monde."

The developments of mathematics are now so great that specialization is a necessity, so that these rival theories need not come into collision; and the pure mathematician may allow the writer on Mechanics to treat of what the name of the subject implies without being compelled to regard his own Geometry as mere Land-Surveying, according to the strict meaning of the word.

There is a tendency in operation among certain mathematicians, as illustrated by Poincaré's remarks on Maxwell's writings, to degrade mathematical argument to mere Calcul, by reducing the experimental facts on which the theory is based to the barest minimum, and that not always clearly established (we venture to instance the Newtonian Law of Universal Gravitation). A vast array of Analysis is in consequence balanced upon a very small amount of axiomatic experiment, which in many cases the smallest divergence of experimental fact is sufficient to upset.

We had hoped at the outset that Duchayla's proof of the Parallelogram of Forces had disappeared, never to reappear again, but it unfortunately pops up on p. 16.

Considering that Statics deals with the Equilibrium of Bodies [illegible] would make a great simplification if the word Resultant was abolished, unless when required to mean a single force reversed of a system of equilibrating forces.

In this way a much simpler proof of the Parallelogram of Forces can be constructed, as indicated by Prof. Maxwell in the Mathematical Tripos; and one figure will now serve for all the possible cases arising in the equilibrium of three parallel forces (p. 47).

Again, when the system is in equilibrium, there is no need to introduce the restriction that the bodies are *rigid* (p. 12); the conditions are precisely the same for elastic bodies; but the system having come to rest, the parts are of invariable form. Every structure (the Forth Bridge, for instance) is composed of elastic parts, but the theorems of elementary Statics are still applicable in the investigation of the principal stresses.

Again, by considering balancing couples, the refined theorems concerning the equivalence of couples in the same or parallel planes, and the composition of couples in different planes, are rendered much more convincing.

In accordance with its title of "Analytical Statics," the theorems concerning the composition and equilibrium of forces in space are treated with reference to co-ordinate axes; but Sir Robert Ball's purely geometrical conceptions of the Wrench, Screw, and Cylindroid are introduced, and discussed from a fundamental standpoint.

A chapter on the determination of Centre of Gravity appears in all treatises on Analytical Statics, just as works on Rigid Dynamics begin with a long and tedious chapter on Moments of Inertia: these subjects should form part of the ordinary treatises on Integral Calculus, and so relieve treatises on Mechanics from at least the principal elements of such calculations.

In the application of the Barycentric Calculus to geometry, the author has made a very interesting collection of problems, well calculated to illustrate the power of this method.

The principal theorems of Statics involve profound geometrical argument, and consequently prove difficult to the majority of students, whose proclivities are usually analytical; but in the applications to Catenaries the analytical interest comes again to the front. Considering that the hyperbolic functions can now be obtained tabulated numerically—for instance, in a table by Mr. T. H. Blakesley, published by the Physical Society—it is curious that the author does not employ them in the discussion of the ordinary Catenary, where their use introduces great elegance and simplicity into the analysis. The figure of the Catenary on p. 316 might with advantage be redrawn, so as to exhibit accurately the principal properties of this curve.

Again, in Example 6, p. 352, where the problem of the catenary is discussed under a central attraction or repulsion, varying inversely as the square of the distance, when the hyperbolic functions are used in conjunction with the circular functions, we are able to write the equation of the catenary in the form—

$$l/r = 1 + \sec\alpha \cos(\theta \sin\alpha), \quad \text{or } 1 + \operatorname{sech}\alpha \cosh(\theta \sinh\alpha),$$

including all possible cases; and it is a curious geometrical result that if these curves are rolled on a straight line, the pole will always describe a circle.

The treatment in § 500 of the catenary curve formed by an elastic rope can also be rendered more elegant by the introduction of the hyperbolic functions.

The chapter relating to Catenaries is headed "Strings." But *string* is used only for tying up parcels; we use a *rope* or *chain* in full scale mechanics, and *thread* in a model; the word *thread* should be used when its own weight is to be neglected, and the words *rope* or *chain* when applied to a true catenary.

A short chapter on Graphical Statics is very welcome, and might with advantage be further developed; and the final chapter, on Machines, is of the usual academic character. The interest of this chapter would be much increased if the diagrams, particularly of the Balance and of the Differential Pulley were taken from objects actually in existence.

The author never employs the absolute units of force, the *poundal* or *dyne*, which he has defined in Chapter I., but works throughout with the gravitation unit. This is in accordance with the universal practice; and to satisfy legal and commercial requirements, these absolute units would require to be defined through the intermediate of the gravitation unit, by taking them as one-gth part of the tension of a thread supporting a pound or gramme weight, the value of g being determined from pendulum experiments. There is no apparatus in existence by which the theoretical definition of the poundal and dyne, derived from dynamical phenomena, could be tested with any pretence to accuracy.

The dyne is the unit of force in the C.G.S. system, but it is a great pity that the commercial units, the metre and the kilogramme, were not adopted; the unit of energy would then be the *joule*, and the unit of power the *watt* or *volt-ampere*. Merely, apparently, for the purpose of making

$$W = sV, \text{ instead of } 1000sV,$$

the Committee of the British Association recommended these niggling C.G.S. units; but considering that for ordinary substances, metals, &c., variations of texture render it unnecessary to tabulate densities beyond four significant figures, the factor 1000 is a positive advantage in numerical calculations, as $1000s$ may be replaced by a whole number.

The "Analytical Statics" is a completely new work, but Dr. Routh's "Dynamics of Rigid Bodies" has been the text-book in universal use for thirty years or more, a better testimony to its merits than anything that could be said here.

It is a pity that a sufficient working knowledge of the simple ideas of Moment of Inertia is not given in a course of the Integral Calculus, so that the author might start immediately on some familiar problems of the motion of a body which turns as well as advances, and relegate the bulk of Chapter I. to a later chapter, when the motion of bodies in space is considered. This long chapter at the outset chokes off many students, who would be encouraged if the principles were introduced in smaller doses, and only as required. The gentlemanly knowledge of this subject, as Maxwell called it, which does not go beyond motion in a plane, is a very valuable mathematical training, and few students go beyond this stage.

D'Alembert's Principle is historically important, as a first clear statement of the mode of forming the equations of motion; but now, in accordance with the modern principle of considering the Third Law of Motion, "Action and Reaction are equal and opposite," as defining a stress composed of two equal opposite balancing forces, D'Alembert's Principle should now be merely looked upon as a convenient mode of writing down the equations of Dynamics in an analytical statical form, when stated in the words, "The reversed effective forces and the impressed forces form a system in equilibrium," while "the molecular, cohesive, or internal forces form a system in equilibrium among themselves."

The much-abused word "centrifugal force" still survives, and need not cause confusion if used to denote the normal component of the reversed effective force of a body moving in a curve.

Early methods of argument in Dynamics were very similar to what we now employ in Thermodynamics, in the statement of the Second Law.

Sir George Airy's commentary on D'Alembert's Principle, quoted on p. 52, forms a very curious contrast to the corresponding explanation in Maxwell's "Matter and Motion."

It would be a strange skeleton frame that Sir George Airy would have had to create to propagate the attraction between the Earth and the Moon or Sun; and an interesting subject of speculation arises as to the modification of Newton's Law of Universal Gravitation when the inertia of the skeleton frame became appreciable.

The discussion on the Pendulum is very complete; Kater's pendulum is fully described, but we miss the account of Repsold's pendulum. In this pendulum the effect of the drag of the air is eliminated by making it symmetrical in shape, but unsymmetrical in density. A short account of Repsold's pendulum will be found in the Account of the Great Trigonometrical Survey; but the pendulum is obviously looked upon with suspicion by our officers, as being employed by their Russian rivals on the other side of the Himalayas.

The very perfection of the pendulum as a method of determining g is the cause of its defect as a means of recovering the standard of length, so that equally skilled observers would differ to an appreciable extent if set to work to reconstitute the standard yard from the seconds pendulum; the clause in the Act of Parliament defining the length of the seconds pendulum is in consequence superfluous.

There is something mysterious and unconvincing in § 109, on the "Oscillation of the Watch Balance"; considering that the inertia of the spring itself is neglected, it seems that the final equation of oscillation might well be written down immediately, without the introduction of any approximation.

The Ballistic Pendulum and its theory are fully described; but it should be pointed out that the pendulum in which the gun itself is mounted gives very untrustworthy records, as the effect of the blast of the powder and of the air dragged along with it is so very great. The Ballistic Pendulum is still useful for determining the velocity of small-arm bullets, but for artillery purposes the electric chronograph has completely supplanted it.

Chapter IV. discusses Motion in Two Dimensions, and is perhaps the most generally important and interesting chapter in the book. A complete dynamical terminology is still a desideratum, and many new words must be coined; for, as De Morgan remarks, "We cannot wait for words, because Cicero did not know the Differential Calculus (or Dynamics)." At the same time it is a pity that the old word *Vis Viva*, meaning Mv^2, was not allowed to drop, to be replaced by *Kinetic Energy*, for $\frac{1}{2}Mv^2$. *Vis mortua* is forgotten as the name for *Work*, and *vis viva*, as the other manifestation of energy, should go too.

The dot notation of Fluxions has been introduced in places: this, though easy to write, is difficult to print, and is inconvenient sometimes with tall letters, while others, like *i* and *j*, are already in their "dotage."

Dr. Routh would, in our opinion, make the working of the illustrative examples more clear, if he always followed the fundamental principle of taking moments about the centre of gravity, as if it was a fixed point: very few students can be trusted to apply the principle to moments about any other moving point, and the equations of relative motion on p. 178 are better kept out of sight of all but a select few.

Dr. Besant's treatment of questions on Initial Motion is in our opinion simpler of application and quite as rigorous as that given in § 199.

A very good collection of illustrative examples completes this chapter, but we miss the extension of the problem of the motion of a cylinder rolling down an incline to the case of a wheeled carriage or of a railway train, when the rotary inertia of the wheels is taken into account, including the determination of the proper position of the coupling chains and buffers; also the investigation of the stresses in the interior of a swinging body like a ship, not only in causing cargo to shift, but also in its physiological bearing on sea-sickness. An ordinary swing is useless as an antidote to sea-sickness, as the seat is close to the centre of oscillation. To feel the disturbing effect we must mount up above the axis of revolution; and to the deck and up the mast of a ship.

As interesting applications, we may mention the dynamics of billiards, §§ 179–98, and of the quintain in § 178.

After Chapter IV. the author launches off into dynamics in space, and now the difficulty of the subject is more than doubled.

Chapter VII., on Energy (or Vis Viva, as Dr. Routh still prefers to call it), precedes in importance and idea the Chapter VI., on Momentum, and might well change place. The idea of energy as $\frac{1}{2}Wv^2/g$ very soon received a name for its unit in the *foot-pound*, but the corresponding name for the momentum, Wv/g, of *second-pound* is as yet hardly known.

In this chapter the Principles of Dynamical Similitude are discussed. In Geometry the Principle of Similitude asserts that a theorem is true whatever the scale on which it is drawn; but in Dynamics the principle is much more complicated, and great care is required in arguing from the performance of a model or of a machine to one to be constructed to a larger scale. The subject is one of great importance at the present time in the discussion of the design of steamers intended to reduce the time of passage across the Atlantic to something under six days; and the statement of the laws to be applied as affecting steamers, first clearly laid down by Mr. Froude, might well find explanation and illustration at this point.

The impact of two rough elastic ellipsoids is treated in §§ 315, &c., by a mathematical *tour de force*; but the expression *perfectly rough* is never met with outside a Cambridge mathematical treatise. What would be the state of things, for instance, between two bodies in contact, one *perfectly rough* and the other *perfectly smooth?* When we wish to produce this so-called perfect roughness between two bodies, we cut teeth on them, to engage together; and in railway travelling the perfect smoothness of the road due to the employment of wheels must be capable of being turned into roughness by the application of the breaks: the continuous breaks now fitted to express trains have enabled a higher average speed to be maintained.

The General Equations of Motion of Lagrange and Hamilton, discussed in Chapter VIII., are not to be employed by any but very advanced students: the formation of these equations and the conversion of one form into the other constituting difficult and refined applications of the Change of the Variables.

In the case where some of the co-ordinates are absent, this part of the subject has received valuable development from Dr. Routh, by means of a principle now called the Ignoration of Co-ordinates.

The volume concludes with an investigation of the Small Oscillations of a System, important as a Stability Test; in such problems the author expresses the result very concisely by means of the length of the simple equivalent pendulum which synchronizes with the oscillations. An interesting problem to discuss is the theory of Mr. Yarrow's Vibrometer, employed for measuring the vertical vibrations of his torpedo-boats: a platform suspended by springs is found to preserve a constant level, if the free period of the vertical oscillations of the platform is incommensurable with the period of the vibrations of the boat.

It is difficult to know where to stop in writing of treatises such as these two of Dr. Routh, so full of detail and interest; and the two treatises together would provide nearly a year's work for an industrious student, who would thereby derive a thoroughly sound and complete knowledge of the subjects.

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COLLECTIONS FROM THE ANDES.

Supplementary Appendix to Travels amongst the Great Andes of the Equator. By Edward Whymper. (London: John Murray, 1891.)

THOUGH many travellers in new or little-known regions, who are not naturalists, have been in the habit of collecting to some extent the more remarkable